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B.A./B.Sc. III Semester Examination

CBS-IIIS/1(P)

21211

MATHEMATICS

Course No.: UMTTC-301

Time Allowed-2 1/2 Hours Maximum Marks-80

Note : Attempt all questions from Sections (A) and (B), each question of Section A and Section B is of 4 marks and 8 marks respectively where as each question of Section C is of 18 marks.

SECTION-A

1. Let x and y be two real numbers such that x > 0. Show that there exist n ∈ N such that nx > 0.

2. Define Cauchy sequence. Show that Cauchy sequence is bounded.

3. Discuss the convergence of the series. $\sum_{n=0}^{\infty} t^n$ Where t > 0.

4. Define uniformly continuous function. Give an example of uniformly continuous function.

5. Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{n x}{1+n^2x^2}$ is point wise convergent.

SECTION-B

1. Let $x_n = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{n^2}$ for n ∈ N. Prove that the sequence (x_n) is increasing and bounded and hence convergent. https://www.jktopper.com

2. Show that there exist an irrational number between two real numbers and hence there exists infinitely many irrational numbers between two real numbers.

3. Let $\sum_{n=0}^{\infty} x_n$ be a positive term series such that $\lim_{n\to\infty} \frac{x_n+1}{x_n} = l$ Show that the series is convergent if $l < 1$ and divergent if $l > 1$

SECTION-C

(Do any two)

1. a) Show that countable union of countable sets is countable. Then show that N × N is countable.

b) Find the supremum and infimum of the following sets

i) S={sin x – 2 cos x: x ∈ R} and

ii) T = {m +$\frac{1}{n}$ =: m, n ∈ N}

2. a) Let (x) be a sequence of real numbers such that

$$\lim_{n\to\infty} x_n = l$$

Then show that $\lim_{n\to\infty} \frac{x_1+\cdots+x_n}{n} = l$

b) Let $\{x_n\}$ be a sequence of real numbers such that

$$\lim_{n\to\infty} \frac{x_n+1}{x_n} = l$$

Where $|l| < 1$. Then show that $\lim_{n\to\infty} x_n = 0$

c) Prove that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{1/n} = e.$

3. Test the convergence of the following series:

i) $1 + \frac{4x}{5} + \frac{4.6x^2}{5.7} + \frac{4.6.8x^3}{5.7.9} + \cdots$

ii) $\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{(2n)!}$

iii) State and prove Weierstrass M-test for uniform convergence of a series of functions $\sum_{n\to\infty} f_n(x)$

i) Show that the sequence $\{f_n(x)\}, where\ f_n(x) = \frac{\sin nx}{\sqrt{n}}$ is uniformly convergent on [0, π].

ii) State and prove Cauchy's condensation test for the convergence of series.

iii) Determine the radius of convergence of the series

$$\sum \frac{(n+1)}{(n+2)(n+3)} x^n$$